NOTATION

U, light energy density; Φ, initial light energy distribution; ρ, distribution function of the random displacements; l, length of a lens; D, distance between lenses; α, specific convergence of the structure in a lens; σ_0, unit 2×2 matrix; σ_i, Pauli matrix; λ, random displacements of the lenses; σ, the variance of the random displacements; N, number of lenses in the light guide.

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CALCULATING ANGULAR RADIATION COEFFICIENTS
BY THE METHOD OF FLOW ALGEBRA

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A method is described for the calculation of mean angular radiation coefficients in two-dimensional systems consisting of any number of plane surfaces, including systems in which two adjacent surfaces form a concave part of the contour. It is shown that for the calculation it is sufficient to know the coordinates of all zone boundaries and the characteristic point of the system.

In calculations of radiative heat exchange between surfaces of a system infinitely stretched out in one direction (a two-dimensional system), the method of flow algebra is widely used for the determination of mean angular radiation coefficients [1, 2]. This method is often called the method of stretched strings, the enveloping curves method, and the algebraic method. In conformity with the notation in Fig. 1, the angular coefficient between two terminal surfaces F_1 and F_2 is given by the simple algebraic expression

$$\varphi_{1,2} = \frac{(AC + BTD) - (AD + BTKC)}{2\,AB} \,, \tag{1}$$

where AC, BTD etc., are the lengths of the elastic strings stretched between the corresponding boundaries of the surfaces F_1 and F_2.

It should be noted that the determination of the lengths of elastic strings in systems with a large number of zones, particularly in the case of calculations with many variants, gives rise to fundamental difficulties and, as a rule, necessitates the use of a computer. Here it is desirable to describe the system by a minimum number of initial values and to calculate the elements of the matrix of angular radiation coefficients according to a universal relation.

The objective of the present work is application of the method of flow algebra for the calculation of angular coefficients in two-dimensional systems of plane surfaces.
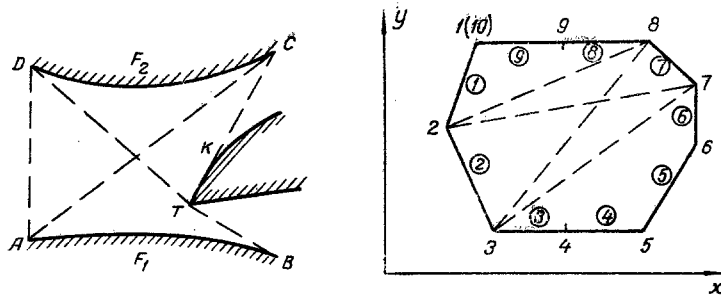
Fig. 1   Fig. 2

Fig. 1. Calculating the angular radiation coefficient by the method of flow algebra.
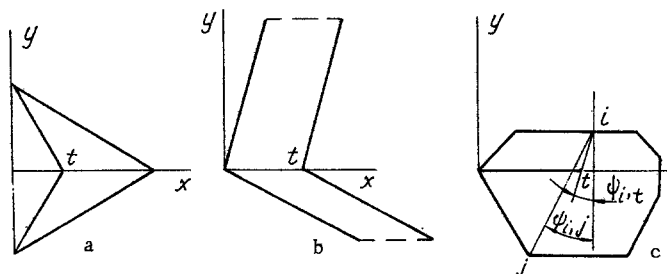
Fig. 2. System without concave parts.

Fig. 3. Examples of systems with one concave part.

§ 1. We consider a closed system consisting of m plane surfaces and not having concave portions. The contour of the system is divided into n zones, with $n \geq m$ (Fig. 2). We number in succession the points corresponding to the boundaries of the zones and the zones themselves so that the i-th zone is located between the points i and i+1. The mean angular radiation coefficient between the zones i and j according to (1) is given by the expression

$$\varphi_{i,j} = \frac{d_{i,j} + d_{i+1,j+1} - d_{i,j+1} - d_{i+1,j}}{2 d_{i,i+1}} . \tag{2}$$

All distances between the zone boundaries constitute the lengths of the straight line segments

$$d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} . \tag{3}$$

The coordinates of zone boundaries in an arbitrary coordinate system are calculated in terms of the geometrical parameters of the system under investigation or are determined by another method, for example, by a graphical method.

§ 2. The approach just described can be extended to a more complex case. Let in a system of plane surfaces two certain adjacent surfaces form a concave part of the contour. This part is an obstacle to the rays between certain zones of the system (Fig. 3). The point of intersection of these surfaces is called the characteristic point of the system. The stretched strings between the boundaries of zones in this system have the form of either straight lines (in the absence of an obstacle) or broken lines of two segments with the break the characteristic point (in the presence of an obstacle).

The system being investigated must be arranged relative to a rectangular coordinate system so that one of the axes (for example, the x axis) divides the system into two parts, in each of which there are no obstacles between the zones. For this it is sufficient for the surfaces forming the concave part to be located on different sides of the x axis and the characteristic point, consequently, to lie on this axis (Fig. 3).

For zone boundaries located in different parts, i.e., having different signs of the coordinate y, the values of the two angles are given by

$$\psi_{i,j} = \mathrm{arctg} \frac{x_i - x_j}{|y_i - y_j|} , \tag{4}$$

$$\psi_{i,t} = \text{arctg} \frac{x_i - x_t}{|y_i - y_t|} .\tag{5}$$

Comparing the values of these angles we can determine the presence or absence of an obstacle between the points being considered, with the comparison condition depending on the direction of the x axis. If the section of the system with the x axis is located toward increasing x from the characteristic point (Fig. 3a and c), then the presence of an obstacle between the points i and j is given by the condition

$$\psi_{i,j} > \psi_{i,t}.\tag{6}$$

In the case where the section is located toward decreasing x from the characteristic point, we use the condition

$$\psi_{i,j} < \psi_{i,t}.\tag{7}$$

The length of the stretched string between the points, when the condition (6) or (7) is fulfilled, is calculated as the sum of the two segments

$$d_{i,j} = d_{i,t} + d_{t,j}.\tag{8}$$

The distances from zone boundaries to the characteristic point and between zone boundaries in the absence of an obstacle are determined according to the relation (3).

The method proposed for the determination of the lengths of stretched strings is universal for systems of such type, i.e., it does not depend on the geometry of the actual system and the number of zones, and this is an important factor when investigating radiative heat exchange in the system while certain geometrical parameters vary.

After the lengths of stretched strings between all zone boundaries have been determined, the mean angular radiation coefficients are calculated according to (2), where by the method just indicated it is sufficient to determine one half of the angular coefficient matrix. For the calculation of the second half of the matrix we use the reciprocity property [1]

$$\varphi_{j,i} = \varphi_{i,j} \frac{d_{i,i+1}}{d_{j,j+1}} .\tag{9}$$

Thus, for the calculation of mean angular radiation coefficients in the systems considered, by the method of flow algebra, it is possible to set up a fairly simple universal calculation program for a computer of any type. For actual systems we have merely to determine the coordinates of all zone boundaries and the characteristic point.

NOTATION

$\varphi_{i,j}$, mean angular radiation coefficient from zone i to zone j; d, length of the stretched string between zone boundaries; x, y, coordinates; t, characteristic point of the system; $\psi_{i,j}$, angle between the direction from point i to point j and a straight line parallel to the y axis; $\psi_{i,t}$, angle between the direction from point i to the characteristic point of the system and a straight line parallel to the y axis.

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